J. Fluid Mech. (1992), *vol.* 236, *pp.* 477–495
Printed in Great Britain

Report of a Symposium on Mechanics of Fluidized Beds

By G. M. HOMSY[1], R. JACKSON[2] AND J. R. GRACE[3]

[1] Department of Chemical Engineering, Stanford University, Stanford, CA 94305, USA

[2] Department of Chemical Engineering, Princeton University, Princeton, NJ 08544, USA

[3] Department of Chemical Engineering, University of British Columbia, Vancouver V6T 1Z1, Canada

(Received 30 September 1991)

Preamble

Fluidized beds are widely used in industry for carrying out a variety of chemical reactions and physical processes. Applications are frequently impeded by a lack of fundamental understanding of the mechanical behaviour of fluidized beds. Despite intensive experimental and theoretical study over the last four decades, there are still many aspects of fluidized beds and related fluid–particle systems that remain obscure. Further work is needed to understand interactions between the particles, influence of particle physical properties, development of non-obtrusive experimental techniques, and study of high-velocity beds and of novel-geometry beds in which particulate solids interact with interstitial fluid.

An international symposium was held at Stanford University on 1–4 July 1991 to discuss recent developments and the current state of knowledge and understanding of the mechanical behaviour of fluidized beds and related fluid–particle systems. The symposium was sponsored by the International Union of Theoretical and Applied Mechanics, and co-funded by the US Department of Energy, National Science Foundation and Electric Power Research Institute. The symposium was attended by 58 specialists representing academic institutions, industry and government research organizations in 11 countries. The diversity of background, coupled with differences in approach, ranging from purely theoretical to fully experimental, led to interesting exchanges where participants were often groping to understand the viewpoint of those involved. The result was frequently rewarding, occasionally perplexing, but certainly stimulating of thought and encouraging for further meetings of this nature.

The scientific committee for the Symposium were G. K. Batchelor (Cambridge University), J. J. H. Brouwers (Trent University), J. Gibilaro (University College London), J. R. Grace (University of British Columbia), G. M. Homsy (Stanford University) (Chairman), R. Jackson (Princeton University), R. I. Nigmatulin (Moscow University) and W. Schneider (Techische Universität Wien).

Each session began with an invited talk for one hour. This was followed by a series of 20-minute presentations. Participants were able at the end of each session to give brief (5 minute) unscheduled mini-presentations. Except for the latter, abstracts were submitted for the presentations, compiled by the organizers and distributed to participants. The meeting also included an informal workshop and a series of video and cinephotographic presentations not reported here. No formal proceedings of the meeting are being published; instead, this report is intended to summarize key findings and areas of discussion.

The titles of all formal presentations are identified with asterisks in the list of References. The report is organized around common themes, and the sequence of presentations may not be that in which they were given. The authors have endeavoured to interpret and extract the essence of the presentations, and apologize if any of the presentations have been misinterpreted or overlooked.

1. Continuum models, rheological behaviour and governing equations

R. Nigmatulin (Moscow University), in an invited talk, began by giving a broad perspective of research on multiphase systems in the Soviet Union. A general treatment of volume-averaging followed, including a careful discussion of the interpretation of the 'particle pressure' as a gradient interfacial force. Balance equations for the mean-square particle velocity fluctuations were also written, and a detailed discussion of microscopic mechanisms by which fluctuations might couple to the mean fields was given. Particular attention was paid to the Magnus force that might exist on the microscale at high particle Reynolds numbers, and how fluctuations in particle angular velocity couple through such a force to produce fluctuations in the linear velocity.

Steady one-dimensional solutions to the resulting equations were also discussed. It is found that there exist two solutions, one in which the mean-square velocity fluctuations are zero, and another in which they have finite constant values, dependent upon the void fraction. Steady one-dimensional solutions with vertical variations associated with stress distributions near a free surface were also presented.

R. di Felice (Universita di l'Aquila) discussed the general problem of describing the macroscopic behaviour of bidispersed mixtures. He considered models in which the mixture of the smaller particles and the fluid is treated as a single 'pseudo-fluid' with effective properties, with the larger particles treated as the dispersed phase. With the proper mixing rules to obtain the effective properties of the pseudo-fluid, it was hoped that two-phase correlations could then be used with success. Two problems, namely the sedimentation of large particles in a suspension and in a fluidized bed, were treated in this fashion, with accurate predictions of the drag resulting in both cases.

H. Buggisch (Universität Karlsruhe) gave an invited lecture about the mechanisms by which stress is generated in a dry, cohesionless granular material. For slow shearing, particle contacts of significant duration form a continually changing network of 'force paths' through which stress is transmitted. Not all the particles belong to this network at any time and, as the rate of deformation increases, the proportion of particles outside the network increases and they oscillate with greater vigour. Thus, to an increasing extent, stress is transmitted by collisions. At low shear rates, whether the stresses are associated with sliding friction, or with alternating local expansions and collapses of the force-transmitting network, Coulomb's law results. At high shear rates, where collisions dominate, there is a cascade of energy from its production in the rate of working of the stress, through the kinetic energy of oscillatory motion measured by the 'granular temperature' (Lun *et al.* 1984), and finally to true thermal energy through dissipation as a result of the inelasticity of collisions. In this case both shear and normal stresses in plane shear are proportional to the square of the shear rate.

The nature of the motion of granular material in the neighbourhood of a bounding wall was next examined, on the basis of experimental results from two different devices: a ring shear cell, with a shoe riding in an annular trough containing the material; and a Couette shear cell in which a 'pseudo-granular material', consisting of rods parallel to the axis of rotation, could be sheared. In each case there was found to be a layer adjacent to the wall, a few particle diameters thick, within which the rate of shear was higher than elsewhere, and where the particles tended to arrange themselves in ordered layers parallel to the wall. With a mixture of particles of different sizes, the larger ones were found to migrate into this layer and, similarly, with a mixture of spheres and cylinders, the cylinders predominated there.

Finally, a solution of Haff's (1983) equations of motion was described for fully developed flow under gravity between vertical confining walls, with the granular temperature constrained to vanish at the walls. Then, with elastic particles, the granular temperature was high near the centreline, but when the particles were highly inelastic both temperature and rate of shear were low across most of the width, with rapid shear confined to narrow layers near the walls, rather like the observations described above for shear cells.

In the discussion there were questions regarding the mechanism of stress transmission through the 'force paths' at low shear rates and the mechanism of segregation by size under shear. It was also pointed out that explicit solutions of the Haff equations are available for plane shear without gravity (Hui *et al.* 1984) and for flow under gravity down an inclined plane, in addition to the case described here, and that the boundary condition of vanishing granular temperature at the wall should probably be replaced with alternatives to be found in the recent literature (Hui *et al.* 1984; Richman & Chou 1988).

The theme of shear layers adjacent to walls was also taken up by D. M. Hanes (Florida University) who applied the collisional stress theory of Lun *et al.* (1984) to a semi-infinite region occupied by granular material, with a horizontal upper surface supporting a plate to which normal and tangential stresses N_0 and S_0 could be applied. Then the corresponding stresses N and S elsewhere in the material are given by

$$S = S_0, \quad N = N_0 + mg,$$

where mg is the weight of a vertical column of material of unit cross-section between the point in question and the upper bounding surface. Profiles of bulk density, granular temperature and velocity within the shearing material were found by solution of the equations of motion. Clearly, shearing can persist only to a finite depth, since the tangential stress remains constant while the normal stress increases with depth, and three criteria were tested for locating the interface between shearing and non-shearing material: zero shear velocity; zero granular temperature; and $S/N = \tan\theta$, the Coulomb yield condition. These led to different depths, but in each case the thickness of the shear layer increased with increasing elasticity of the collisions between particles. For related earlier work, see Hanes & Inman (1985*a*, *b*).

C. Thornton (Aston University) showed the results of some direct simulations of the initiation of flow of spherical grains from a hopper. As soon as the hopper exit was opened, a wave of stress relaxation moved up through the material at sonic speed in the granular matrix, leaving most of the material in the middle of the hopper in an almost stress-free condition. Significant stresses were confined to material close to the bounding wall. The downward motion then appeared to develop very nearly under the influence of gravity alone. There was, therefore, reason to question the relevance of the familiar treatments of hopper flow, which are based on plasticity models for the stress in the flowing material.

O. Walton presented some results from dynamic simulation of a complete inclined chute, with three-dimensional particles fed into its upper end and moving down under gravity. For fully developed, shallow flowing layers it was found that the bulk density decreased monotonically on moving up through the layer from the base of the chute to the free surface, and the velocity increased monotonically with height at a rate which was largest at the bottom of the layer. In contrast, for thick, fully developed layers, the bulk density was lowest at the base of the chute. On moving up through the flowing layer it rapidly increased to an asymptotic value, which was then retained up to the free surface. Correspondingly, shearing was confined to the thin layer of reduced bulk density adjacent to the base of the chute, while the region of constant bulk density above slid as a block. The larger the value of the coefficient of restitution for collisions between particles, the greater the overall thickness needed before a non-shearing layer appeared. These results seem to be consistent with the predictions of theories of fully developed chute flow based on continuum equations of motion.

J. D. Goddard (University of Southern California) treated the mechanics of equilibrium and slow deformation in granular materials in an invited lecture. He first reviewed some basic ideas and history of the mechanics of granular materials, with attention to contributions of Reynolds, particularly the famous observations of dilatancy. The whole range of behaviour from elastic deformation, through plastic yield to Coulomb behaviour under continued slow deformation was traced, followed by a discussion of observed rate dependence of the stress at higher rates of deformation. Most of the remainder of the presentation was devoted to two problems, one dealing with deformations so small that the assembly of grains behaved elastically, and the other with plastic yield and Reynolds dilatancy under large strains.

Micromechanical models of granular materials based on Hertzian contacts predict that the incremental elastic moduli of the assembly of particles vary as $p^{\frac{1}{3}}$ under changes of pressure, whereas many experiments show a $p^{\frac{1}{2}}$ dependence. The speaker attributed the discrepancy to structural changes that result from inelastic 'shake down' associated with mechanical disturbances to which the material has been exposed, and a percolation model of this phenomenon has been proposed (Goddard 1990).

For large strains the phenomenon of Reynolds dilatancy may be regarded as an internal constraint which relates the volume of a body of material to its shape. Direct computer simulation of assemblies of cylindrical particles under large strains indicate that they asymptotically approach the geometric percolation threshold, so this is to be identified with the 'critical state' of Schofield & Wroth (1968).

Finally, these ideas were combined with effects due to the presence of a fluid in the pores of a saturated soil to generate speculation on the mechanism of 'seismic liquefaction'. This might be regarded as a shape-induced fluidization associated with dilatancy, and it was speculated that liquefaction could correspond to the mechanical percolation threshold. A major part of this presentation is covered in a recent publication (Bashir & Goddard, 1991).

C. Campbell gave a brief unscheduled presentation regarding some experiments in which thermal conductivity and shear stress were simultaneously measured in a shear cell for different solids volume fractions. The dimensionless conductivity and shear viscosity were found to follow quite different trends with solid concentration, with heat and momentum transfer being of different orders of magnitude.

2. Fluid–particle interactions, instabilities and waves

In his invited talk, G. K. Batchelor (Cambridge University) surveyed what is rigorously known about the interactions between particles and fluid, and between particles themselves, with the ultimate goal of arriving at equations of motion based on firm principles and understanding of the micromechanics. He began by recounting that many of the difficulties, for Stokes flow at least, are related to the slow spatial decay of disturbances caused by the presence of particles. Interaction among many particles may be summed by 'renormalization' techniques to give results for the drag or hindered settling function in the dilute regime that may be considered to be small-ϕ expansions of empirical laws, such as the Richardson–Zaki expression. These expressions may be extended to bidispersed systems reasonably satisfactorily, and can be exploited to explain the occurrence of spatial instability in the sedimentation of such systems.

Attention was then focused on fluctuations and their effect on macroscopic behaviour. The divergence of fluctuations with spatial extent in a statistically homogenous suspension was recalled, and three plausible sources of cutoff (container size, finite inertia, and gradient dispersion) were put forth.

Consideration of the origins of particle stress focused on the combined effect of momentum flux due to particle velocity fluctuations and a repulsive interaction due to gradient dispersion as the main contributors to the 'particle pressure' or bulk elasticity in a fluidized suspension. He then recounted the results of his paper (Batchelor 1988) on the stability of fluidized beds to one-dimensional waves, showing stability if the bulk elasticity of the particle configuration is sufficiently large. He closed by suggesting further tests of the equations, which included particles raining from the roof of a wall-slug (as discussed by P. Agarwal and G. Wallis, see below), and an interesting buoyancy-driven secondary instability of a steady travelling train of one-dimensional waves. Unfortunately, time did not allow a full description of his recent results on the latter problem.

In the first part of his invited talk, R. Jackson (Princeton University) posed the intriguing question of whether the idealized fluidized bed, i.e. one of uniform expansion and no particle motion, actually exists in practice. In search of the 'elusive idealized bed', he began with this working definition of such a bed and observed that the equations of motion in which the particulate stress is treated as both isotropic and reversible have mathematical solutions which successfully describe it. Whether it exists in practice is related both to the assumptions about the nature of the stress carried by particles and the stability of the ideal state to (small) fluctuations. With the exception of the recent work by Ham *et al.* (1990), the most commonly quoted possibility for such a state is the observation that gas-fluidized beds of fine particles do not bubble immediately above U_{mf}.

Following an amusing set of quotations from a series of authors, all of whom claim to have done the definitive linear stability analysis, Jackson reviewed such analyses, leading to the general conclusion that viscous resistance to motion leads to preferred wavenumbers of instability and that any physical factors which result in an effective 'particle pressure' or bulk elasticity can lead to a range of stable operation.

He then described a series of meticulous experimental measurements, both macroscopic (pressure drop–flow rate) and microscopic (local void fraction variations in the axial direction), for gas fluidization of FCC particles. These measurements yielded hysteresis loops in the pressure drop–flow rate curves, with bubbling at $U_{mb} > U_{mf}$ occurring when the hysteresis loop closes. Accompanying these observations were direct measurements of structural variations within the bed in the expanded state, leading to the conclusion that for these fine particles, the ideal uniformly expanded bed does not exist. A mechanical interpretation of the results was given in terms of a finite tensile yield stress of non-hydrodynamic origin, leading to structures that become tenuous and fragile near U_{mb}.

In a paper discussing models that might explain experimental observations of stable beds, such as that of Ham *et al.* (1990), J. Jenkins (Cornell University) discussed statistical-mechanical approaches that have analogues in granular media on the one hand and statistical physics of dense systems on the other. In searching for mechanical descriptions of stabilizing terms, such as hydrodynamic dispersion and bulk elasticity, he argued for the inclusion of a field equation for fluctuating quantities, as is common in rapid-granular-flow theory. On the other hand, he also suggested that a phase transition, similar to that of hard-sphere systems at high densities, provides an alternative explanation of experimental findings.

D. Koch (Cornell University) presented a kinetic-theory analysis of the motion of particles in an attempt to derive expressions for the Reynolds stress (momentum flux) carried by fluctuations in particle velocity as a function of Stokes numbers, *St* (see also Koch 1992). Two limits of $St \gg \phi^{-\frac{3}{4}}$ and $St \ll \phi^{-\frac{3}{4}}$ were analysed. In general, the resulting expression indicates a *non-local* effect. It was concluded that such effects cannot be modelled simply as a particle pressure, at least in the parameter range for which the theory applies. In the discussion that followed, it was commented that momentum fluxes might differ from pressure gradients, that the Reynolds stress is probably a stabilizing contribution to the bulk elasticity, and that the treatment of fluctuations did not seem to include a prediction of gradient diffusivities.

P. Singh (University of Minnesota) presented some experiments and theory claiming that the ill-posedness of some models of linear instability of a uniform bed may be 'regularized' by taking the finite particle size into account (see also Singh & Joseph 1992). By a geometric argument, the relationship between area and volume fraction was established, and the continuity equation for area fraction derived. This was seen to be different from that for volume fraction, and, in addition, leads to the aforementioned regularization. Experiments on very dilute, 'two-dimensional' beds (only slightly wider than a single particle diameter) were described. Digital imaging and spectral analysis of measured fluctuations in the area fraction demonstrated a spectral structure that mimics the so-called 'blockage function' which relates area and volume fractions.

Discussion of this work was vigorous and prolonged, with little resolution of what seemed to be questions of both interpretation and applicability of continuum theories that predict phenomena on scales of order one particle diameter.

P. Agarwal (University of Adelaide) described a series of experiments on 'square-nosed slugs', i.e. slugs which fill the entire cross-section of an apparatus. It was argued that their rise velocity, in contrast to that of round-nosed slugs, is limited by the rate at which particles rain from the roof. Except for some intriguing 'stick–slip' phenomena, this velocity was observed to be constant. Quantitative measurements show that the velocity is independent of vessel size, but dependent on particle size. Interpretation was made in terms of (gradient) hydrodynamic dispersion causing particles to fall from the roof, and values of the dispersion coefficient were thus inferred. Although the experiments cover a vastly different range of void fraction, the results were seen to be in reasonable agreement with a bold extrapolation of those of Ham & Homsy (1988) and Davis & Hassen (1988).

G. Wallis (Dartmouth College) considered the same problem of particles falling from the roof of a slug in one-dimensional motion, but with a more general physical mechanism in mind, namely that particles experience a force, dependent upon gradients of void fraction, that causes them to accelerate away from the interface. By envisioning the roof as a shock in particle concentration and employing one-dimensional models, Wallis considered the effect of different model hypotheses on the predictions of wave velocity *vs.* particle flux. Contrary to previous work (including some by Professor Wallis himself!), a model in which the force is proportional to $d^2\epsilon/dz^2$ was seen to agree far better than one in which the force is equivalent to a finite bed compressibility, i.e. proportional to $d\epsilon/dz$. No physical interpretation was given of the origin of a force with this dependence on void-fraction gradient.

Y. Buyevich (Urals State University) presented a wide-ranging talk, detailing results that have been in the Soviet literature for some time, but (regrettably) have been relatively unknown in the West. His main theme was to establish several properties of the macroscopic behaviour of fluidized beds and, more generally, fluid–particle systems from the point of view of statistical mechanics. As a mathematical physicist, he drew on many powerful techniques of statistical physics, based on a representation of fluctuations in terms of their Fourier transforms. This, together with assumptions about the microstructural details, allows unified and *a priori* calculation of many transport quantities, such as the effective viscosity of a suspension, the 'hindered settling' function (or equivalently the Richardson–Zaki equation) for sedimentation and fluidization, and the self-diffusion tensor in sedimentation. Material not presented because of insufficient time deals with Chapman–Enskog expansions, Percus–Yevick/Starling equations of state for dense systems, and the relationship between bed elasticity and the stability of fluidization.

Two papers discussed the mathematical solution properties of a set of reasonable continuum equations with the objective of establishing some of the qualitative (bifurcation) features of solutions. The first, by Dankworth & Sundaresan, presented by S. Sundaresan (Princeton University) discussed one-dimensional solutions to a set of fully nonlinear dynamical equations for fluidized beds. Although simplified, the set captured many of the desirable features of a model, including the existence of solutions describing the 'ideal' bed (in the sense of Jackson) and linear instability points. Although algebraically complex, the bifurcation analysis of these equations indicates that as U increases, solutions describing weakly non-linear waves quickly reach double heteroclinic connections, leading to slug-like solutions with regions of nearly constant voidage separated by sharp transitions. The dilute states are nearly particle-free, leading to speculation that the heteroclinic connection signals the onset of slugging. Some further speculations were given regarding two- or three-dimensional instability of slug-like solutions, through a homoclinic connection, giving rise to turbulent beds.

M. Goez (Universität Heidelberg) described a bifurcation analysis of a set of equations similar to those of Sundaresan, but in more than one dimension. In addition to the one-dimensional nonlinear travelling wave, the analysis reveals the existence of two-dimensional travelling oblique waves, similar to those observed in liquid fluidized beds. It was speculated that there is a further evolution to two-dimensional standing travelling waves (stationary in the transverse and travelling in the axial directions) corresponding to the formation of bubbles. If the speculation proves correct, it will provide a definitive link between the instability of the uniform state and the occurrence of bubbles. While such a link has often been assumed and/or thought to exist, it has not yet been proven. This work has been written up for publication (Goez 1990, 1992).

In discussion of both bifurcation papers, the algebraic details of which were daunting even for experts, the main concern seemed to hinge on the sensitivity of such detailed calculations to specific underlying modelling assumptions.

Several presentations were concerned with inhomogenities in the interior of a deforming granular material, rather than shear layers adjacent to walls. R. P. Behringer (Duke University) showed X-ray movies (taken at 10 frames/s) of sand discharging from a wedge-shaped hopper. When the sand grains were angular in form these revealed a regular sequence of bands of reduced bulk density propagating through the flowing material. The direction of propagation depended on the hopper angle: for angles larger than 40° the bands moved upward from the discharge slot, while for angles smaller than 40° they moved downward. If the angular sand was replaced with sand whose grains were smooth, no such bands were seen, and bands could not be generated by mixing smooth sands of different grain sizes. However, with mixtures of smooth and angular sand there was found to be a critical fraction of the angular material, above which the bands were seen during discharge. Clearly, therefore, the phenomenon is related to the roughness of the grains. The findings are described in detail elsewhere (Baxter *et al.* 1989; Baxter, Leone & Behringer 1991, 1992). In an attempt to understand this effect theoretically, rough grains were replaced by elongated grains in a cellular automaton model of the flowing material, and this was claimed to give a good description of the physical flows.

In separate experiments power spectra of traces of the fluctuating normal stress at points on the wall of a discharging hopper were generated, and were found to be of the form $P \propto \omega^{-\alpha}$, where α varies between 1.4 and 2.1. Surprisingly, there is no feature of the power spectrum that can be identified with the characteristic timescales of the system.

A possible explanation of Behringer's observations may be found in a stability analysis described in the presentation by D. G. Schaeffer (Duke University) (see also Lu & Schaeffer 1991). This was based on a plasticity model of proper frame-indifferent form, which included anisotropic hardening. For perturbations about a state of homogeneous deformation, the linearized equations have solutions of the form $\exp ik(\boldsymbol{n}\cdot\boldsymbol{x}-ct)$. If c is real these represent stable propagating waves with wave vector in the direction of the unit vector $\boldsymbol{n}$, while for imaginary c the disturbances are unstable standing waves, which are thought to represent the initiation of stationary shear bands. For complex values of c the disturbances have the form of propagating waves which grow or decay as they move, and in the unstable case they are said to represent a 'flutter instability', which may correspond to the initiation of moving bands such as those observed by Behringer. It was found that the flutter instability, if it existed, was confined to a bounded interval of the total shear strain $\gamma_1 < \gamma < \gamma_2$, measured from the initiation of motion, while the shear band instability occurred for all sufficiently large values of the strain, $\gamma > \gamma_3$.

S. B. Savage (McGill University) (see also Savage, 1992) again addressed the question of inhomogenities in flowing granular material from the point of view of a stability analysis, but in this case it was applied to a material in continued, rapid deformation, which could be described by a collisional model such as that of Lun *et al.* (1984). First he noted that fluctuations are inherent in flowing granular materials, given the discrete nature of the system, and showed an example of a trace of the torque on the shoe of a ring shear apparatus driven at constant speed of rotation. Despite the fact that the trace represented the integrated effect of the shear stress over the whole circumference of the shoe, it showed marked fluctuations, and the stress on a more local area of contact with the shearing material would undoubtedly show even larger fluctuations. These observations are not surprising, since fluctuation theory suggests that their magnitude should be proportional to $1/n^{\frac{1}{2}}$, where n is the number of particles in the characteristic volume, which is very large for typical molecular systems but much smaller in experiments on granular materials.

Recent computer simulations of uniformly shearing granular materials (Hopkins & Louge 1991) have revealed the presence of quite large 'clusters', or regions of increased bulk density, that form and dissipate continually in the shearing material, which raised the question of whether continuum equations of motion would predict instability in uniform shearing. This question was examined using the equations of Lun *et al.* (1984) to describe the motion. The perturbations considered were plane waves whose wavenumber vector rotated in conformity with the rotation associated with the plane shearing, and only linear stability was considered. Nevertheless, despite the fact that the base state was spatially uniform, the algebra was heavy. It was found that waves of sufficiently short wavelength were damped, but longer waves were unstable, with the longest axial waves growing most rapidly, implying a vertically striated mode of instability. In the dynamic simulations of Hopkins & Louge periodic conditions are imposed, and these limit the spatial extent of predicted disturbances. However, it was found that the size of the clusters observed increased with the size of the periodic cell, and that cluster formation was suppressed completely if the cell was too small. These observations are seen to be consistent with the results of the continuum stability analysis.

J. Kok (Twente University) described investigations of the velocity and attenuation of waves generated by an oscillating paddle at the upper surface of a 0.6×0.6 m square fluidized bed of 300 μm polystyrene spheres. For frequencies below 1.6 Hz, wave velocities agreed well with the simple well-known result for small-amplitude surface waves in deep liquids, as predicted by Needham (1984). Attenuation of these waves was also consistent with what would be expected for a liquid having a kinematic viscosity of about 0.01 m^2/s, a value which is of the same order as expected from measurements of Schugerl, Merz & Fetting (1961). When the wave generation frequency was increased above 1.6 MHz, the waves became very dispersive, the propagation velocities increased and there was a sharp transition in attenuation character. The extent to which this transition is related to limitations in the experimental apparatus is unclear.

Transmission of disturbance waves through fluidized beds was addressed in the paper by Filla *et al.* which was presented by S. Vaccaro (Universita di Napoli). Periodic and impulsive disturbances were generated in beds operated just below and just above the minimum bubbling velocity, using the time delay of pressure signals reaching two points to calculate the propagation velocity. These measured velocities were usually between 10 and 25 m/s. They are reasonably well predicted by a simple analysis due to Davidson (1991), whereas an elastic model for the dense phase employing fluidized-bed rheological data gave large errors.

P. Salatino (CNR, Napoli) described an extensive investigation of minimum fluidization and minimum bubbling conditions for a variety of particles fluidized by CO_2 at pressures up to 85 bar. These are particularly valuable since they carry the particle/fluid density ratio into the intermediate range between gas and liquid fluidization, where few data are presently available. The particles used were 95 and 175 μm glass ballotini, 76 μm stainless steel spheres, and 450 μm 'amberlite' spheres. The last had a density of only 660 kg/m^3. As the pressure was changed the fluid density varied from 5 to 600 kg/m^3, while the viscosity changed by a factor of only four.

Qualitatively, it was found that the interval between the gas velocities for incipient fluidization and minimum bubbling increased rapidly with increasing fluid density, as might be expected. Foscolo & Gibilaro (1984) gave an estimate of the void fraction at minimum bubbling, based entirely on fluid-mechanical forces exerted on the particles, and containing no adjustable parameters. This model was tested against the above observations and was found to underpredict the minimum bubbling void fraction. Salatino also asserted that the void fraction for minimum bubbling might be expected to depend only on the density ratio, and the Froude number for a single particle at its terminal velocity, if the forces stabilizing the bed are entirely hydrodynamic in origin. However, the set of experimental observations was found to be inconsistent with the existence of such a relation. It was tentatively concluded that other forces, such as direct interactions between particles, must be involved.

In discussion it was questioned whether a stability criterion can be used to locate the initiation of bubbling. It was also suggested that particle–particle interactions, if significant, might be detected by the footprint they leave in the form of hysteresis between fluidization and defluidization curves.

3. Jets, spouted beds and bubble formation

There has been considerable work on jets of gas injected into fluidized beds in connection with the performance of distributors, quenching of hot gases, etc.

Presentations by J. A. M. Kuipers (Twente University) and by H. S. Caram (Lehigh University) were both concerned with the process of bubble formation by injection of a flow of gas locally into a uniformly fluidized bed. In order to predict the rate of growth of such a bubble it is necessary to know how much gas leaks from the growing cavity into the surrounding bed and simple, semi-empirical models, such as those of Davidson & Harrison (1963) and Zenz (1971), either neglect this leakage or equate the leakage velocity at the surface of the cavity to the minimum fluidization velocity.

Kuipers (see also Kuipers, Prins & Swaaij 1991) considered a two-dimensional fluidized bed, with local supplementary injection of gas through the distributor to form the bubble. Their calculation of bubble growth rate was based on a direct numerical solution of equations of motion for the gas and particle phases, interacting through a drag force. The momentum equation for the particles included both a particle phase pressure and dissipative stresses, and the solution was generated on a minicomputer. From computed contours of bulk density the boundary of the bubble was defined as the contour corresponding to void fraction 0.85, and this served as the basis for computed values of horizontal, vertical and equivalent bubble diameters, and the gas leakage rate. Results of the computations were compared with measurements on a two-dimensional bed of 500 μm diameter particles of intrinsic density 2660 kg/m^3, at minimum fluidization conditions, with various rates of injection of gas to form the bubble. The profile of the computed bubble at various times was in remarkable agreement with photographs of the growth of the physical bubbles. A plot of the measured equivalent diameter, as a function of time, was also predicted with remarkable accuracy, in contrast to the two simple models referred to above, which both grossly underestimated the rate of leakage of gas during the early stages of growth.

This work is important as one of the first instances of a careful comparison of a solution of equations of motion for a fluidized suspension, *in more than one dimension*, with experimental observations. In response to questions, Kuipers indicated that the computations had also been extended to follow the rise of the bubble through the bed after detachment from the air supply orifice, and that the spontaneous development of bubbles from a small perturbation of a uniform bed had been investigated.

Caram studied a three-dimensional bubble formed by injection of a constant flow of gas into an interior point of an infinite, uniform fluidized bed. Attention was restricted to the initial stages of growth, before any appreciable upward motion due to gravity. During this time the bubble could be regarded as a sphere centred on the point of gas injection, and the problem was one-dimensional. The equations of motion for gas and particles could then be reduced to just two ordinary differential equations for the particle velocity and the void fraction, which took into account particle inertia, drag forces exerted between the gas and the particles, and a particle phase pressure. They were solved by a numerical finite-difference method, and the solutions were compared with observed rates of growth of bubbles in a semi-cylindrical fluidized bed. These were generated by injecting gas at a constant rate through a tube whose opening was immediately adjacent to the flat wall of the bed, thus giving rise to hemispherical bubbles that could be observed through the wall. The predicted relation between dimensionless bubble diameter and dimensionless time was found to be insensitive to the value of the single parameter left after scaling, namely a dimensionless particle phase elasticity. Agreement with experimentally measured growth rate was good, with slight underestimation by the theory of the rate of gas leakage. An interesting feature of the theoretical solutions, not shown in the two-dimensional predictions of Kuipers discussed above, was the prediction that gas leakage reverses after some initial interval; in other words, gas is sucked back into the growing bubble, rather than expelled. However, the spherically symmetric solution is valid only during the interval before bubble rise is significant, so this effect may not be observable.

In a paper by Zierfuss & Schneider, presented by W. Schneider (Technische Universitaẗ Wein), supplemented by a video film (see also Zierfuss & Schneider 1992), an analysis was carried out of a steady vertical jet using a relatively simple modelling approach. One-dimensional approximations were applied to the jet region, while an apparent fluidized bed viscosity and hydrostatic pressure gradient were incorporated for flow in the surrounding dense phase region. A friction coefficient at the jet interface, described by means of jump conditions, was the only fitted parameter. Reasonable predictions of jet shapes and jet penetrations were said to be given by the model.

In spouted beds (Epstein & Grace 1984), gas (or sometimes liquid) enters through a single central orifice at the bottom of a vessel which usually has an inverted conical shape in the lower part. A variant is a spout–fluid bed where most of the gas enters through a single central orifice, but there is also additional auxiliary air added around the primary orifice to keep the solids in continuous motion. Cylindrical draft tubes are sometimes used to stabilize the spout (jet) in spouted and spout–fluid beds. These features are combined in a reactor being developed at the University of Calgary for rapid pyrolysis of heavy oil (Muir, Berruti & Behie 1990; Stocker, Eng & Behie 1990). A cold model (200 mm diameter semi-cylindrical) version of this reactor was the subject of a paper by Milne *et al.* presented by L. Behie and of an accompanying video presentation. It was shown that mass fluxes of the order of 200–1000 $kg/m^2\,s$ solids could be delivered up the draft tube, with negligible clustering or axial dispersion. The addition of the auxiliary gas helped augment the flow of particles as well as providing an excellent means of control. However, increased temperature made the central jet less stable, and an alternative configuration has therefore been explored with particles drawn from the periphery into a draft tube that extends right to the bottom. This configuration appears to be attractive, with good control over the solids flux in the tube by varying the flow of fluidizing fluid to the bed of solids which surrounds the central draft tube.

4. Bubbling fluidized beds

In his invited lecture J. R. Grace (University of British Columbia) examined the widely held view that fluidization is, in some sense, smoother and better for materials with a wide distribution of particle sizes. This is at odds with the well-known Geldart classification, which depends only on the mean particle diameter and the densities of the solid and fluid phases. Much of the work has been reviewed in a recent paper (Grace & Sun 1991).

To test the effect of size distribution, three different materials with the same surface/volume mean particle size were prepared by elutriation of FCC catalyst, followed by controlled mixing of the fractions obtained. Two of the materials had smooth size distributions, one wider than the other, while the third had a bimodal distribution. Each type of material was prepared from both spent and fresh catalyst, separately. Experiments then fell into two groups, the first designed to test the effect of size distribution on various aspects of the mechanics of fluidization, and the second to examine its effect on reactor performance.

Two tests are commonly used to assess the aeration of the dense phase, namely a collapse test, in which the time of collapse of a fluidized bed is measured after the gas flow is suddenly cut off, and an expansion test, which determines the width of the interval of bubble-free fluidization beyond minimum fluidization. The material of widest size distribution was found to have the longest collapse time and the widest interval of bubble-free fluidization, confirming the popular view of the influence of size distribution on aeratability. In separate tests it was also found that, for a given gas flow in the bubbling range, the material with the widest size distribution gave the smallest bubbles, and hence the greatest overall bed expansion. With the material of wide size distribution there was also a significantly larger concentration of particles within the bubbles, and these were predominantly from the smaller end of the size distribution.

It is often of technical importance to expand the bed beyond the regime of bubbling into turbulent fluidization, and it was found that this could be achieved at lowest gas flow for the material of widest size distribution.

Studies of reactor performance were made by activating the catalyst for the ozone–oxygen conversion reaction. Then, it was found that for a given mass of catalyst loaded in the bed, the measured conversion at each value of the gas velocity was largest for the material with the widest size distribution. Studies were also made in which, for a given overall catalyst activity, the fine particles were activated more strongly than the coarse ones, and vice versa. Conversion was then found to be higher when the activity was more concentrated in the finer particles. This is, perhaps, a consequence of the above finding, that the finer particles tend to concentrate within the bubbles.

A technique for tracking single radioactive tracer particles in fluidized beds was described by M. Chen (University of Illinois) (see Lin *et al.* 1985). A scandium-coated glass bead of 1 to 2 mm diameter is detected by an array of 16 sensors deployed around the vessel. A close estimate of the instantaneous position is then obtained by calculation from the radiation intensity at each of the sensors. Differentiation with time then leads to estimates of the local velocity. Ensemble averaging over extended periods of time provides a wealth of information regarding such features as flow patterns and mixing characteristics in the bed. While the technique has limitations in terms of accuracy, it has provided unique data on bubbling beds. The data handling techniques have recently been improved. In addition, a new fluidized Couette flow apparatus has been constructed to allow a range of measurements of shear, velocity profiles and rheological models which could be helpful in understanding the mechanics of fluidized solids.

C. S. Campbell (University of Southern California) described a device (see also Campbell & Wang 1990, 1991) designed to measure the forces exerted on the wall of a fluidized bed by the particles, rather than the fluidizing fluid. It consists of a flexible diaphragm whose displacement measures the difference between the normal forces exerted on its two sides. Vent holes, too small to permit the passage of particles, balance the pressure of the fluid in a closed cavity behind the diaphragm with that of the fluid in the bed, so that the net force can be attributed entirely to the presence of the particles on the side exposed to the bed. In a more recent version of the device the diaphragm has been replaced by a screen, which is everywhere permeable to fluid but retains the particles within the bed.

Measurements were represented by plotting the time-average particle pressure, at a given location on the bed wall, as a function of the fluidizing fluid velocity. Typically, the pressure first decreased as the gas flow increased from zero, reaching a sharp minimum at the condition of incipient fluidization, after which it increased and approached an asymptotic value corresponding to fully developed slugging in the bed. For given values of the gas flow below minimum fluidization the particle pressure decreased on moving up the bed, while for flows above minimum fluidization it increased.

Curves for particles of different sizes could be brought close together by plotting the particle pressure against mean void fraction, rather than gas velocity, and many measurements could be correlated by scaling the average particle pressure with the product of the estimated maximum bubble size and the weight of a single particle.

Traces of particle pressure as a function of time consisted of a sequence of peaks, correlated with the passage of bubbles, separated by intervals of relatively low pressure. The peaks were attributed to the inertia of surges of particles thrust toward the wall during the passage of each bubble. However, in discussion an alternative mechanism was suggested, whereby the convective motion due to the bubble first dissipated into an increase in granular temperature, which is then perceived as particle pressure at the wall.

Another aspect of forces exerted on solid surfaces by bed particles was discussed by R. Clift (University of Surrey), who was concerned with the forces on horizontal tubes immersed in the bed. Bubbles rising through the bed were found to be attracted toward a tube so that they collided with its lower surface, separated, then reformed above the tube. This process was sometimes accompanied by a marked displacement of the bubble in the direction of the axis of the tube. The attraction of bubbles to the tube was attributed to the form of gas streamlines near it. These separate to pass round the tube, leaving a stagnation point on its lower surface, so the vertical component of gas velocity is small in a region just below the tube. Consequently particles drop out of suspension in this region, leaving a cavity almost free of particles on the lower surface of the tube. This, in turn, provides a low-resistance path for gas flow, so the gas streamlines are attracted toward it, and the gas rising through the bed flows preferentially toward the tubes. The bubbles then follow the track of these streamlines.

As each bubble passes round a tube, the particles rising in its wake impact the lower surface of the tube, causing an impulse of duration 10–100 ms, after which the force on the tube falls back to an almost constant value as the bubble slides round it. Finally, as the bubble moves away, the force drops to a background value attributed to the burden of defluidized material resting on the upper surface of the tube.

Following an illuminating film showing fluidized bed combustion of coal particles, J. Brouwers (Twente University) presented a model for reaction of spherical char particles in a fluidized bed for two limiting cases of small and large Thiele modulus so that there was kinetic control or burning in an advancing surface layer, respectively. The results were recast and combined to give an approximate expression for burnout times of single particles. The burning of volatiles in the dilute freeboard of fluidized bed combustors was treated by G. van der Honing (Twente University) (see also van der Honing 1991). Allowance was made for decay of turbulence and macro- and micromixing by means of a model due to Spalding (1971).

5. Circulating fluidized beds and risers

When the gas velocity across a column containing particles is increased sufficiently, the distinction between a dense 'bed' and freeboard region disappears and the system operates in the fast fluidization regime. While there is substantial entrainment, the column can still be operated with a volume fraction of solids of typically 5–10% by recapturing the entrained solids continuously in cyclones or other gas–solid separation devices and returning them to the base of the column (often called a 'riser') through a seal of some kind, often consisting of a standpipe and mechanical or non-mechanic valve. This system, consisting of the riser and associated external recirculation loop, is referred to as a circulating fluidized bed. There are a number of significant applications, chiefly in combustion of a wide variety of fuels in a clean and efficient manner and in solid-catalyzed reactions like hydrocarbon cracking. This has encouraged substantial research activity (e.g. Basu 1986).

J. Werther (Technical University Hamburg-Harburg) gave an invited lecture about recent experimental and modelling work on mixing and motion of gas and particles in circulating fluidized beds. After a brief review of the literature, he presented data showing solids flux measurements obtained using a suction probe, which could be aimed upwards or downwards in order to measure downward flux or upward flux separately. These data were obtained in a large (8 m diameter) coal combustion circulating fluidized bed unit. Data in such large units are rare and therefore valuable. These data confirmed findings in much smaller units showing that there is a substantial region near the wall where the particle flow is predominantly downwards, this reverse flow helping to maintain a dense suspension in the riser despite the high gas velocities (typically of order 8 m/s) and high net solids flux (typically 15–100 kg/m^2 s). These results suggest that the wall-layer thickness is proportional to the diameter or width of the unit.

Werther then presented new data on gas mixing obtained by injecting carbon dioxide tracer into a cold model experimental unit of diameter 0.4 m about 5 m above the base. Gas was then sampled at other positions, upstream as well as downstream, in order to disengage the two unknown parameters in a two-region mixing model which visualizes the flow structure as consisting of a dilute upflowing core and a denser annular downflowing wall layer, with exchange between the two regions. The thickness of the wall layer is typically 10 or 15% of the column radius. Surprisingly, the data indicated that the radial mixing in the dilute core was insensitive to the concentration of particles in that region.

In a model directed at predicting the hydrodynamic behaviour of circulating fluidized beds, K. Wirth (Universität Erlangen) (see also Wirth 1991) envisaged a series of dense cylindrical strands or clusters extending over the entire height of the riser, with transfer of momentum by impaction of solid particles, each collision dislodging another particle. The dilute phase was assumed to consist of gas and widely dispersed particles, with a relative velocity equal to the terminal settling velocity of single particles. There is only one fitted constant in the model. Solution of the model for given particles and gas led to a plot of dimensionless pressure drop against particle Froude number, with a volumetric flow ratio as parameter. This plot predicts coexistence of a denser lower region at higher gas flows.

A different kind of model was proposed by G. Jansen (Royal Dutch Shell), this one oriented towards risers operated with finer (70 μm) catalysts and higher gas velocities (15 m/s). Cluster formation as well as collisions were ignored. A numerical model based on interpenetrating continua was derived utilizing mass, momentum and energy conservation equations. Turbulence was modelled, with the aid of k,ϵ-closure relationships, based on a kinetic model of dilute hard spheres. The model, like that of Tchen (1947), accounts implicitly for the added-mass and Basset history terms. Typical spectra were presented, but no full predictions were offered at this time. It is intended that the model will allow predictions in large industrial risers, using the experimental results of van Bruegel, Stein & Vries (1969) to test its validity.

Modelling of heat transfer under pneumatic conveying conditions was performed by M. Louge (Cornell University) in work which is an extension of a paper by Louge, Mastorakos & Jenkins (1991) dealing with the corresponding hydrodynamics. The model makes use of momentum and kinetic energy equations for the gas, with corresponding momentum and granular temperature equations for the particles. Boundary conditions include application of the law of the wall to the gas at the edge of the turbulent core and a slip condition for the particles. Good agreement has been demonstrated with the experimental results of Tsuji, Morikawa & Shiomi (1984).

Thermal energy equations were written in terms of the heat transfer for both gas and particles, with coupling provided by a term, sensitive to particle diameter, for transfer between these two phases. In the boundary conditions, it is assumed that the particles themselves transfer negligible heat directly to the wall. The model was shown to be capable of predicting Nusselt numbers for experimental systems where the addition of particles first leads to a decrease in heat transfer due to suppression of turbulence followed by an increase due to exchange with the gas, causing a steeper temperature profile near the outer wall.

In a brief unscheduled presentation, M. Louge described some intriguing experimental results where addition of a very thin surface layer to 110 μm glass beads led to substantially different axial suspension density profiles in a circulating fluidized bed. The influence of the surface change was attributed to friction of particles moving along the wall, but in discussion it was noted that the coefficient of restitution would also have changed so that changes in rebounding of particles during interparticle collisions could be an important contributing factor.

6. Related topics

A. Ladd (Lawrence Livermore National Laboratory) discussed the possibility that 'lattice gas' or 'cellular automota' methods might be adapted to the simulation of multiphase systems. After first reviewing the technique, its adaptability to numerous parallel computers, and its application to molecular fluids, he discussed some recent comparisons between lattice gas and cellular automota for the roll-up of the mixing layer of a Navier–Stokes fluid. He concluded with a discussion of some of the conceptual and computational difficulties associated with the construction of a massive particle using such lattice representations of continua.

U. Felderhof (Technical University Aachen) (see also Felderhof 1991) discussed the problem of the calculation of the virtual mass coefficient in dispersed systems. After quoting some previous results from the literature, including those from so-called cell models, he discussed the solution of the inviscid equations of motion around particles in the limit of high Stokes numbers and dilute dispersions. A connection was made between this problem and other potential problems in effective-medium theory for dielectrics. The final result for the virtual mass coefficient,

$$C(\phi) = (1-\gamma\phi)/(2-3\phi+\gamma\phi),$$

differs from standard cell-model results, and contains a parameter γ which depends upon the spatial distribution of particles.

A. Sangani (Syracuse University) returned to the problem of the dynamics of inclusions in potential flow, a plausible model for bubbly liquids at high bubble Reynolds numbers. He provided a general theoretical description of bubble flow in a three-dimensional periodic box which formed the basis of a three-dimensional numerical simulation technique. An impressive video showing large numbers of non-identical bubbles, produced by animation of the simulations, was shown. Several distinct bubble interaction mechanisms were identified, but the results were very recently obtained and await detailed analysis.

C. Thornton, in the presentation already referred to in §1, included a discussion on the attrition of particles, a problem of technical and practical importance in a wide variety of applications. Large-scale simulations of particle attrition in three dimensions were reported. The particle is modelled as an agglomerate, made up of a very large number of smaller particles held together at contact points by forces of a prescribed magnitude. It was found that a critical impact velocity exists for breakup of the agglomerate. Detailed visual analysis of the time sequence of attrition demonstrates that particle breakup occurs through the propagation of a crack through the agglomerate.

In the discussion that followed, comparisons were made with both crack propagation theory and bond percolation, as discussed earlier by J. Goddard. It was suggested that casting the results in dimensionless terms might help in generalizing them to particle sizes, bond strengths, and impact velocities outside the range of the current simulations.

REFERENCES†

*Agarwal, P. K., Khakhar, D. V., Gururajan, V. S. & Lim, K. S. Raining of particles from an emulsion-gas interface in a fluidized bed. Also, submitted to *Chem. Engng Sci.*

Bashir, Y. M. & Goddard, J. D. 1991 A novel simulation method for the quasi-static mechanics of granular assemblages. *J. Rheol.* **35**, 849–885.

† An asterisk denotes a paper presented at the Symposium. The address of a participant who presented a paper may be obtained from Prof. G. M. Homsy.

BASU, P. (Ed.) 1986 *Circulating Fluidized Bed Technology*. Pergamon.

BATCHELOR, G. K. 1988 A new theory of the instability of a uniform fluidized bed. *J. Fluid Mech.* **193**, 75–110.

*BATCHELOR, G. K. Hydrodynamic interaction of particles and its consequences.

BAXTER, G. W., BEHRINGER, R. P., FAGERT, T. & JOHNSON, G. A. 1989 Pattern formation in flowing sand. *Phys. Rev. Lett.* **62**, 2825–2829.

BAXTER, G. W., LEONE, R. & BEHRINGER, R. P. 1991 Time-dependence and pattern formation in flowing sand. *Eur. J. Mech.* B **10**, 181–186.

BAXTER, G. W., LEONE, R. & BEHRINGER, R. P. 1992 Experimental determinations of time-scales in flowing sand. *Phys. Rev. Lett.* (submitted).

*BEHRINGER, R. P., BAXTER, G. W., LEONE, R. & JOHNSON, G. A. Time-dependence, scaling and pattern formation in flowing sand.

*BROUWERS, J. J. H. Coal combustion in a fluidized bed.

*BUGGISCH, H. Wall effects in granular flow.

*BUYEVICH, Y. A. Fluctuations and dispersion in fluidized beds.

CAMPBELL, C. S. & WANG, D. G. 1990 A particle pressure transducer suitable for use in gas-fluidized beds. *Measurement Sci. Technol.* **1**, 1275–1279.

*CAMPBELL, C. S. Particle pressures in gas-fluidized beds.

CAMPBELL, C. S. & WANG, D. G. 1991 Particle pressures in gas-fluidized beds. *J. Fluid Mech.* **227**, 495–508.

*CARAM, H. S. & PIERRAT, P. 1992 Bubble formation and gas leakage in beds at minimum fluidization conditions. Also, submitted to *Fluidization VII Conf.*

*CHEN, M. M., SUN, J. G. & HOANG, T. Radioactive particle tracking measurements of particle dynamics in gas fluidized beds.

*CLIFT, R. Forces on horizontal tubes in fluidized beds.

*DANKWORTH, D. C. & SUNDARESAN, S. Time-dependant flow patterns arising from the instability of uniform fluidization.

DAVIDSON, J. F. 1991 The two phase theory of fluidization: successes and opportunities. *AIChE Symp. Series*, vol. 87, No. 281, pp. 1–12.

DAVIDSON, J. F. & HARRISON, D. 1963 *Fluidized Particles*. Cambridge University Press.

DAVIS, R. H. & HASSEN, M. A. 1988 Spreading of the interface at the top of a slightly polydisperse sedimenting suspension. *J. Fluid Mech.* **196**, 107–134.

EPSTEIN, N. & GRACE, J. R. 1984 Spouting of particulate solids. In *Handbook of Powder Science and Technology* (ed. L. Otten & M. Fayed), chap. II. Van Nostrand-Reinhold.

*FELDERHOF, B. U. Virtual mass in two-phase flow.

FELDERHOF, B. U. 1991 Virtual mass and drag in two-phase flow. *J. Fluid Mech.* **225**, 177–196.

*FELICE, R. DI A pseudo-fluid model to describe the behaviour of binary-solid suspensions.

*FILLA, M., MASSIMILLA, L., MUSMARRA, D. & VACCARO, S. Propagation velocities of disturbances originated by gas jets in fluidized beds. Also, submitted to *Intl J. Multiphase Flow*.

FOSCOLO, P. U. & GIBILARO, L. G. 1984 A fully predictive criterion for the transition between aggregative and particulate fluidization. *Chem. Engng Sci.* **39**, 1667–1675.

GODDARD, J. D. 1990 Nonlinear elasticity and pressure-dependent wave speeds in granular media. *Proc. R. Soc. Lond.* A **430**, 105–131.

*GODDARD, J. D. Reynolds dilatancy, microstructural breakdown and seismic liquefaction in granular media.

*GOEZ, M. F. Bifurcation analysis of fluidized bed equations.

GOEZ, M. 1990 Instabilities and bifurcations in a two-dimensional bed model. *Z. Angew Math. Mech.* **70**, 386–388.

GOEZ, M. 1992 On the origin of wave patterns in fluidized beds. *J. Fluid Mech.* (submitted).

*GRACE, J. R. Influence of particle size distribution on the behaviour and performance of fluidized beds.

GRACE, J. R. & SUN, G. 1991 Influence of particle size distribution on the performance of fluidized bed reactors. *Can. J. Chem. Engng* **69**, 1126–1134.

HAFF, P. K. 1983 Grain flow as a fluid mechanical phenomenon. *J. Fluid Mech.* **138**, 401–430.

HAM, J. M. & HOMSY, G. M. 1988 Hindered settling and hydrodynamic dispersion in quiescent sedimenting suspensions. *Intl J. Multiphase Flow* **14**, 533–546.

HAM, J. M., THOMAS, S., GUAZZELLI, E., HOMSY, G. M. & ANSELMET, M.-C. 1990 An experimental study of the stability of liquid-fluidized beds. *Intl J. Multiphase Flow* **16**, 171–185.

*HANES, D. M. The thickness of a collisional, granular, shear flow in a half-space with gravity.

HANES, D. M. & INMAN, D. L. 1985*a* Observations of rapidly flowing granular-fluid materials. *J. Fluid Mech.* **150**, 357–380.

HANES, D. M. & INMAN, D. L. 1985*b* A dynamic yield criterion for granular materials. *J. Geophys. Res.* **90**, 3670–3674.

*HONING, G. VAN DER Bubble initiated turbulent mixing above fluidized beds.

HONING, G. VAN DER 1991 Volatile and char combustion in large scale fluidised bed coal combustors, Ph.D. thesis, Twente University.

HOPKINS, M. A. & LOUGE, M. Y. 1991 Inelastic microstructure in rapid granular flow of smooth disks. *Phys. Fluids* A **3**, 47–57.

HUI, K., HAFF, P. K., UNGAR, J. E. & JACKSON, R. 1984 Boundary conditions for high-shear grain flows. *J. Fluid Mech.* **145**, 223–233.

*JACKSON, R. The elusive fluidized bed: does it really exist?

*JANSEN, G. H. & ROMATE, J. E. Modelling of two-phase riser flow.

*JENKINS, J. T. Viscous fluctuations and the fluidization of concentrated suspensions.

KOCH, D. L. 1992 Anomalous diffusion of momentum in a dilute gas–solid suspension. *Phys. Fluids* (submitted).

*KOCH, D. L. & KUMARAN, V. Kinetic theory for gas–solid suspensions.

*KOK, J. B. W. Propagation velocity and rate of attenuation of surface waves on a homogeneously fluidized bed. Also, submitted to *Intl J. Multiphase Flow.*

*KUIPERS, J. A. M., PRINS, W. & SWAAIJ, W. P. M. VAN Theoretical and experimental bubble formation at a single orifice in a two-dimensional gas-fluidized bed. *Chem. Engng Sci.* **46**, 2881–2894 (1991).

*LADD, A. J. C. Dissipative and fluctuating hydrodynamic interactions via lattice-gas cellular automata.

LIN, J. S., CHEN, M. M. & CHAO, B. T. 1985 A novel radioactive tracking facility for measurement of solids motion in straight and tapered fluidized beds. *AIChE J.* **10**, 924–929.

LOUGE, M., MASTORAKOS, E. & JENKINS, J. T. 1991 The role of particle collisions in pneumatic transport. *J. Fluid Mech.* **231**, 345–359.

*LOUGE, M., YUSOF, J. M., JENKINS, J. T. & MASTORAKOS, E. Heat transfer in the pneumatic transport of massive particles. Also, submitted to *Intl J. Heat Mass Transfer.*

LU, L.-J. & SCHAEFFER, D. 1991 The flutter instability in granular flow. *J. Mech. Phys. Solids* (in press).

LUN, C. K. K., SAVAGE, S. B., JEFFREY, D. J. & CHEPURNIY, N. 1984 Kinetic theories for granular flow: inelastic particles in Couette flow and slightly inelastic particles in a general flow. *J. Fluid Mech.* **140**, 223–256.

*MILNE, B. J., BERRUTI, F. & BEHIE, L. A. Gas and particle flow characteristics in the entrainment region of spouted and spout-fluid beds with draft tubes.

MUIR, J., BERRUTI, F. & BEHIE, L. A. 1990 Solids circulation in spouted and spout-fluid beds with draft tubes. *Chem. Engng Commun.* **88**, 153–171.

NEEDHAM, D. J. 1984 Surface waves on a homogeneously fluidized bed. *J. Engng Maths* **18**, 259–271.

*NIGMATULIN, R. I. Continua mechanics and averaging theory for monodisperse gas-particle suspension with a random motion and collision of dispersed particles.

RICHMAN, M. W. & CHOU, C. S. 1988 Boundary effects on granular shear flows of smooth disks. *Z. Angew. Math. Phys.* **39**, 885–901.

*SALATINO, P., POLETTO, M. & MASSIMILLA, L. Stability of uniform gas fluidized beds operated with CO_2 in ranges of pressure and temperature between ambient and nearly critical conditions.

*Sangani, A. S. & Didwania, A. K. Dynamic simulation of bubbly flows at large Reynolds numbers.

*Savage, S. B. Diffusion, stability and segregation in granular shear flows.

Savage, S. B. 1992 Instability of an unbounded uniform granular shear flow. *J. Fluid Mech.* (in press).

*Schaeffer, D. G. The flutter instability in granular flow.

Schofield, A. & Wroth, P. 1968 *Critical State Soil Mechanics*. McGraw-Hill.

Schugerl, K., Merz, M. & Fetting, F. 1961 Rheologische Eigenschaften von gasdurchströmten Fliessbet systemen. *Chem. Engng Sci.* **5**, 1–38.

*Singh, P. & Joseph, D. D. Finite size effects in fluidized beds.

Singh, P. & Joseph, D. D. 1992 Dynamics of fluidized suspensions of spheres of finite size. *J. Fluid Mech.* (submitted).

Spalding, D. B. 1971 Concentration fluctuations in a round turbulent free jet. *Chem. Engng Sci.* **26**, 95–107.

Stocker, R. K., Eng, J. H. & Behie, L. A. 1990 Hydrodynamic and thermal modelling of a high temperature spouted bed reactor with a draft tube. *Can. J. Chem. Engng* **68**, 302–311.

Tchen, C. M. 1947 The motion of small particles suspended in a turbulent flow. Ph.D. thesis, Delft University of Technology, The Hague.

*Thornton, C., Kafui, D. K. & Yin, K. K. Computer simulated agglomerate collisions.

Tsuji, Y., Morikawa, Y. & Shiomi, H. 1984 LDV measurements of an air–solid two-phase flow in a vertical pipe. *J. Fluid Mech.* **139**, 417–434.

Van Bruegel, J. W., Stein, J. J. M. & de Vries, R. J. 1969 Isokinetic sampling in a dense gas–solids stream. *Proc. Inst. Mech. Engrs* **184**, 18–23.

*Wallis, G. B. Decompression waves in fluidized beds.

*Walton, O. Computer simulation of inclined chute flow.

*Werther, J. Particle motion and dispersion of gas in circulating fluidized beds.

*Wirth, K. E. Fluid mechanics of circulating fluidized beds.

Wirth, K. E. 1991 Fluid mechanics of circulating fluidized beds. *Chem. Engng Technol.* **14**, 29–38.

Zenz, F. A. 1971 Regimes of fluidized behaviour. In Fluidization (ed. J. F. Davidson & D. Harrison), pp. 1–23. Academic.

*Zierfuss, R. & Schneider, W. Jet-like flows in fluidized and packed beds.

Zierfuss, R. & Schneider, W. 1992 Jet flows in fluidized beds. *Z. Angew Math. Mech.* (in press).